TECHNICAL EFFICIENCY ANALYSIS OF HOME-VISIT LONG TERM CARE SERVICES IN JAPAN

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Abstract

The home-visit long-term care services (HLCS) market is the only market in Japan where nonprofit, for-profit, and public providers compete against each other for consumers by competition of the quality of their services, as the price is regulated by the long-term care insurance system. The empirical analysis in this paper shows that nonprofit providers produce HLCS most efficiently of the three and that both provider size and service quality have significant impacts on technical efficiency regardless of the service providers' organizational structure. The analysis also shows that expanding size and improving quality ameliorate the technical efficiency of production.

1. INTRODUCTION

Japan is already a rapidly aging society. It is said that one out of three Japanese will be defined as elderly by the middle of the 21st century. The burden on care attendants is much heavier than it used to be because care attendants are also elderly. The Ministry of Health, Labour and Welfare (2002) reported that over 50 % of care attendants are currently 60 years old or over, and long-term care tends to linger for a long time: one of every two bedridden persons is bedridden for three years or more. In addition, the number of women in the workplace is increasing, and therefore it is becoming very difficult for fami-

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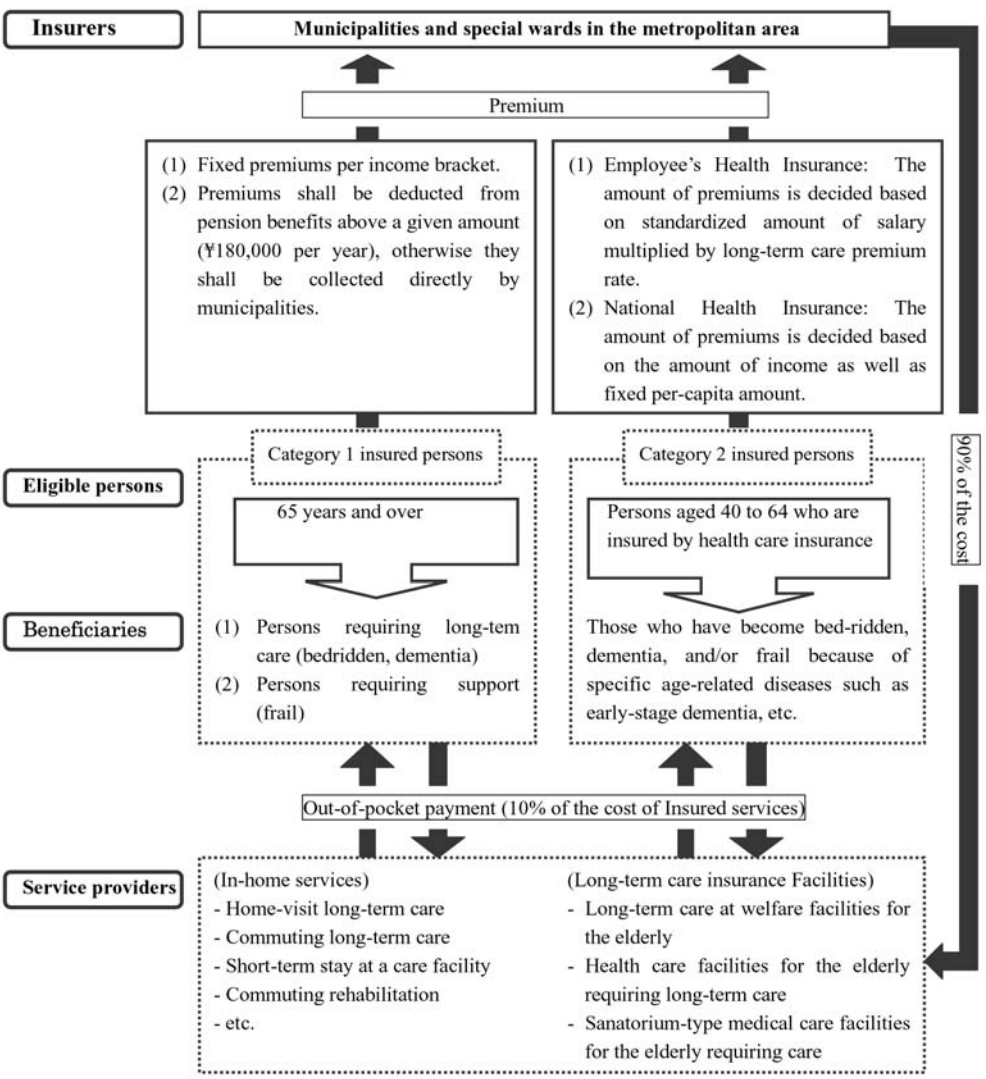
lies to act as the sole caregiver to elderly people.

In order to alleviate the burden of care attendants and the society's major concerns about aging and elderly care, a long–term care insurance system was implemented on April 1, 2000.

Figure 1 shows an outline of the long–care insurance system.

Figure 1: Outline of the long–term care insurance system



Note: Created by the author based on Ministry of Health, Labour and Welfare (2003).

Insurers are municipalities and special wards in the metropolitan area. The central government, prefectures, health care insurers and pension insurers cooperate closely and provide assistance to them. There are two types of insured persons; category 1 (aged 65 or over) and category 2 (aged 40 to 64 who are insured by health care insurance). Benefi-

ciaries, premiums, and methods of levying and collection for the two categories are different.

In order for insured persons to consume long-term care services, first of all, they must obtain long-term care requirement certification issued by the municipality. The municipality issues the requirement certification based on a screening judgment made by the long-term care approval board. The long-term care approval board will examine the mental and physical condition of the insured person and make a screening judgment based on his or her doctor's opinions.

Out-of-pocket payment of care services is 10% of the cost of both insured services, with a standard charge for meals. There is an upper limit of the above out-of-pocket payment. Also, the upper limit of the out-of-pocket payment and standard charge for meals is set lower for people with lower incomes.

Category 1 insured people pay a fixed insurance premium set by each municipality according to their income level. This reduces the burden on people with low income. The insurance premium of people insured under category 2 with Employee's Health Insurance is determined based on a standardized salary multiplied by the long-term care premium rate. Likewise, based on the amount of income as well as a fixed per-capita amount, the insurance premium of people insured under category 2 with National Health Insurance is determined.

This insurance system, rare by global standards, has two major features. One is that the system separates long-term care from medical care insurance. This is appropriate because, unlike the medical insurance system, the long-term care insurance system is a fledgling system, and therefore requires frequent reexaminations of the user's costs and benefits.

The other is that the system is organized so that user-centered and high quality services are efficiently delivered. To be more precise, those who require long-term care can use services of their choice, and welfare services and health and medical care services are provided in a comprehensive and unified manner. Before the long-term care insurance was implemented, only public service providers were allowed to supply long-term care services. This made long-term care services relatively uniform, since the governmental provision of quasi-public goods is predominantly influenced by the preferences of median voters due to the constraint of electoral policies. Such constraint on governmental action means that the quasi-public goods supplied tend to become relatively homogeneous. Consequently, some non-median voters with homogeneous preferences within a heterogeneous population face dissatisfaction from consuming too little of the government provided heterogeneous quasi-public goods. The non-median voters, therefore, seek to discover other suppliers that fulfill their unmet demand for quasi-public goods. According to Weisbrod (1986, 1988), unsatisfied demand will be manifest principally in the for-profit sector and the nonprofit sector. Though nonprofit and for-profit organizations also pro-

duce and supply quasi-public goods, unlike governments however, they do not face the constraint of electoral policies since their supply targets are not median voters but non-median voters. Consequently, in the presence of demand heterogeneity, the nonprofits and for-profits have a comparative advantage over the government sector in producing and supplying heterogeneous quasi-public goods, and the presence of demand heterogeneity is a basic source of government failure.

On the other hand, asymmetric information exists when consumers do not know all that they may care to know with regard to the goods and services they wish to obtain, until after payment has taken place (Ben-Ner 1986; Anheier and Ben-Ner 1997). In general, information regarding goods tends to be biased towards producers since they know more about quality and quantity of the traded goods (Hansmann 1987). This information asymmetry between producers and consumers is the basic source of contract failure. When consumers cannot detect information asymmetry at low cost, they will be reluctant to purchase the goods they need, for fear of being cheated by the profit-maximizing behavior of for-profit producers, who have an incentive to cheat consumers (Young 2001). In particular, it is very difficult for consumers to measure quality of quasi-public goods, which tends to be significantly biased towards producers. Therefore, it is conceivable that the costs associated with information asymmetry of quasi-public goods will become extremely high. When consumers cannot detect information asymmetry at low cost, they will prefer nonprofit organizations to for-profit organizations. Consequently, supplementation of quasi-public goods will tend to be in the public and nonprofit sectors with the for-profit sector being relatively small.

The long-term care insurance system has developed the unprecedented three-sector economy in Japan. Since a variety of independent enterprises such as nonprofit providers and for-profit providers can participate in the long-term care market, nonprofit, for-profit, and public providers compete against each other for consumers. They do so by competition of the quality of their services rather than by means of price, as the price is regulated by the long-term care insurance system. As a result, consumers can now enjoy higher quality, a greater variety, and a greater quantity of services. In fact, Ministry of Health, Labour and Welfare (2003) reported that the usage of the home-visit long-term care services (quasi-public goods) has increased dramatically under this new system. Monthly average usage of the home-visit long-term care services (*HLCS*) was about 3.55 million people in 1999. In May 2001, the number expanded to about 6.45 million people, an 82% increase in demand. Use of the other quasi-public goods such as the commuting long-term care, short stays, and group homes for the elderly with dementia have also increased, but increases in usages of these services were not as dramatic as *HLCS* use. It can therefore be seen that *HLCS* is a core aspect of the long-term care insurance system.

In order to investigate the interrelated roles of the three-sector economy and to grasp the feature of *HLCS* production, we utilize a stochastic frontier analysis. The stochastic

frontier model, originally proposed by Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977), involves an unobservable random variable associated with the technical inefficiency of production for an individual provider, in addition to the random error in a traditional regression model. Within the framework of the stochastic frontier analysis, we focus on the interrelationships of the three factors: the technical efficiency, the provider size, and service quality. In particular, we expect that the service quality is an important factor influencing the technical efficiency of production because nonprofit, for-profit, and public providers compete against each other for consumers by competition of the quality of their services rather than by means of price. They do so because the price is regulated by the long-term care insurance system. In the literature on firm growth, efficiency plays a significant role in the growth of firms. Jovanovic (1982) proposed a model where efficient firms grow and survive whereas inefficient firms decline and fail.

The relationship between the technical efficiency and the firm size in the framework of the stochastic frontier analysis is examined in several relevant empirical studies. For example, Pitt and Lee (1981) and Mengistae (1996) reported that the firm size-efficiency relationship is positive. Brada, King, and Ying Ma (1997) and Lundvall and Battese (2000) also found that the size effects are positive for an overwhelming majority of the firms. Applying the deterministic frontier analysis, Page (1984) and Little et al. (1987) found its positive relationship in some sub-sectors. However, there is no reason to expect that firms of different service quality would operate at different levels of technical efficiency. The quality-efficiency relationship is basically left unexplained in traditional neo-classical economics. Therefore, this paper investigates the relationship between the size-service quality and technical efficiency for *HLCS* providers in three sectors of the economy.

The rest of this paper is laid out as follows. In section 2, we introduce more detail on the stochastic frontier production model applied in this paper and the survey that has been conducted for an estimation of the model in this paper. Section 3 is devoted to policy implications based on the results from the estimations. Conclusions will summarize our findings.

2. THE MODEL

2.1. THE STOCHASTIC FRONTIER MODEL

This paper employs a stochastic frontier production function in order primarily to investigate the interaction of the three factors; provider size, quality of output (the home-visit long-term care service), and technical efficiency. Output is defined as a function of a given set of inputs with technical inefficiency effects, which define the degree to which providers fail to reach the frontier because of technical inefficiencies of production.

Following the recommendation of Battese and Coelli (1993) this paper employs a general specification for the model as a starting point and test for simpler formulations within a formal hypothesis-testing framework. Hence, the stochastic frontier production function is specified here as a translog function. In addition, this model specifies that these inefficiency effects are modeled in terms of other explanatory variables. The stochastic frontier production function is defined as

$$\begin{aligned}\ln y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{3i} + \beta_4 \ln x_{si} \\ + \sum_{k=1}^{3} \alpha_k \ln x_{ki} + \sum_{k=1}^{3}\sum_{j=1}^{3} \alpha_{kj} \ln x_{ki} \ln x_{ji} \\ + \beta_5 DR_{1i} + \beta_6 DR_{2i} + \beta_7 DR_{3i} + \beta_8 DJ_{1i} + \beta_9 DJ_{2i} + V_i - U_i,\end{aligned} \tag{1}$$

where the subscript i indicates the observation for the i-th provider, lns indicate natural logarithms;

y_i represents remuneration units for the long-term care service;

x_{si} represents production technology measured by years in business;

D_{1i} is a dummy variable, which has a value of one if at least one administrative staff was employed, otherwise it is zero;

D_{2i} is a dummy variable, which has a value of one if at least one full-time helper was employed, otherwise it is zero;

D_{3i} is a dummy variable, which has a value of one if at least one part-time helper was employed, otherwise it is zero [1)] ;

x_{1i} is the maximum between the number of administrative staff and $1-D_1$;

x_{2i} is the maximum between the number of full-time helpers and $1-D_2$;

x_{3i} is the maximum between the number of part-time helpers and $1-D_3$;

DJ_{1i} has a value of one if the provider i is a public provider;

DJ_{1i} has a value of one if the provider i is a for-profit provider;

DR_{1i} has a value of one if the provider i is located in the special are or special area A;

DR_{2i} has a value of one if the provider i is located in the area A;

DR_{3i} has a value of one if the provider i is located in the area B. See Appendix I for the area division in detail.

The V_is are assumed to be independent and identically distributed (*iid*) normal random variables with variance, σ_v^2 and mean zero or *iid* $N(0, \sigma_v^2)$;

The U_is are non-negative random variables which are assumed to be independently (but not identically) distributed, such that U_i is the truncation of the normal distribution with variance, σ^2 and non-negative mean, μ_i expressed as

1） Battese (1997) proved that if dummy variables, D_{1i}, D_{2i}, and D_{3i} are not included to account for an intercept change, then the estimator for the responsiveness of $\ln y_i$ to $\ln x_{1i}$, $\ln x_{2i}$ and $\ln x_{3i}$ are all biased.

$$\mu_i = \delta_0 + \delta_1 D_{3i} + \delta_2 \ln z_{1i} + \delta_3 \ln z_{2i} + \delta_4 (\ln z_{1i})^2 + \delta_5 (\ln z_{2i})^2 + \delta_6 (\ln z_{1i} \times \ln z_{2i}) + \delta_7 DJ_{1i} + \delta_8 DJ_{2i} + W_i, \quad (2)$$

where the W_i s are random variables, which are assumed to be identically distributed, and obtained by truncation of the normal distribution with mean zero and variance, σ_w^2 ;

z_{1i} is the size of the provider i, which is measured by the number of part-time helpers;

z_{1i} is the quality of *HLCS* of the provider i.

Pitt and Lee (1981), Mengistae (1996), Little et al (1987), and Brada, King, and Ying Ma (1997) also used a number of workers as a proxy for the firm size. Equation (2) specifies that the means of the normal distributions, which are truncated at zero to obtain the distributions of the technical inefficiency effects, are not the same, but are functions of values of observable variables and a common vector of parameters (truncated normal with heterogeneous mean). The model is obviously a simplification, which does not account for possible correlation structures among random errors (the V_i s), associated with particular firms nor heteroschedasticity in the random errors and the technical inefficiency effects. The log-likelihood function for this stochastic frontier and inefficiency model and its partial derivatives with respect to the parameters of the model are shown in the Appendix in Battese and Coelli (1993). The likelihood function is expressed in terms of the variance parameters, $\gamma = \sigma^2 / \sigma_s^2$ and $\sigma_s^2 = \sigma^2 + \sigma_v^2$.

This paper assumes that desk workers, full-time helpers, and part-time helpers are systematically different from each other in some aspects. Their different skills and hourly wages may support this assumption. Most helpers who actually visit insured persons for long-term care services are part-time helpers, whereas full-time helpers are either care workers, or coordinators who assign part-time helpers to insured persons, or supervisors who train part-time helpers.

The cross terms of the natural logarithm of x_{1i}, x_{2i}, x_{3i} capture the interactions of desk workers, full-time workers, and part-time workers. This paper uses a production input variable as a proxy for provider size. This approach has previously been adopted by two applications of the stochastic frontier production function on agriculture (Coelli and Battese 1996; Ngwenya et al. 1997; Battese and Broca 1997), in which farm size was represented by a function of the input in the inefficiency model. This paper uses the natural logarithm of the numbers of the part-time helpers, x_{3i}, to represent provider size. The choice of this input variable is motivated by the observation that about 77% of total workers in a sample data for estimation in this paper are part-time helpers, and therefore, the natural logarithm of x_{3i} might be an appropriate proxy for the capacity of each sample provider producing the *HLCS*.

The logarithm of service quality is used in order to investigate the relationship be-

tween technical efficiency and quality of services, which reflects outputs of *HLCS* in this paper. It is not obvious whether pursuing high quality of services could worsen or improve the technical efficiency of production. The squares and interaction between provider size and service quality are included to allow for U-shape and joint relationships between the two variables and technical efficiency.

Given the specifications of the stochastic frontier production function, defined by equation (1), the technical efficiency of the i-th provider is defined by

$$TE_i = \exp(-U_i)\,. \tag{3}$$

The program FRONTIER *4.1* developed by Coelli (1994) estimates all parameters and produces predictions for technical efficiency for each provider and the arithmetic average of the predictors for technical efficiencies (Coelli, Parasada Rao, and Battese 1998). The variance parameters are also estimated by *FRONTIER 4.1* in terms of $\gamma = \sigma^2/(\sigma^2 + \sigma_v^2)$, and $\sigma_s^2 = \sigma^2 + \sigma_v^2$.

Various tests of hypothesis of the parameters in the frontier production function and the technical inefficiency models can be performed using the generalized likelihood ratio test statistic, defined by $\lambda = -2\,[l\,(H_0) - l\,(H_1)]$, where $l\,(H_0)$ is the log-likelihood value of a restricted frontier model, as specified by a null hypothesis, H_0; and $l\,(H_1)$ is the log-likelihood value of the general frontier model under the alternative hypothesis, H_1. This test statistic has a chi-square (or a mixed chi-square) distribution with degrees of freedom equal to the difference between the parameters involved in the null and alternative hypotheses.

If the inefficiency effects are absent from the model, as specified by the null hypothesis, H_0: $\gamma = \delta_0 = \delta_1 = \cdots = \delta_8 = 0$, then the statistic, λ, is approximately distributed according to a mixed chi-square distribution. In this case, critical values for the generalized likelihood ratio test are obtained from Table 1 in Kodde and Palm (1986). If this null hypothesis is true, then the production function is equivalent to the traditional average response function that can be efficiently estimated using ordinary least-squares regression.

Due to the squared and interaction terms on the right-hand side in the translog stochastic frontier production function given by equation (1), the elasticity of output with respect to inputs are functions of the levels of the inputs. Since the number of part-time helpers also appears in the technical inefficiency model given by equation (2), the output elasticity with respect to this input variable is a function of the values of the inputs in both the production frontier and the technical inefficiency models. According to Battese and Broca (1997) the general expression for the input elasticity of the mean output with respect to the k-th input for provider i is given by

$$\frac{\partial \ln[E(y_i)]}{\partial \ln x_k} = \left(\beta_k + 2\beta_{kk} + \sum_{j\neq k}^{3} \beta_{kj} \ln x_{ji}\right) - \Psi_i\left(\frac{\partial \mu_i}{\partial \ln x_k}\right), \quad k = 1,2,3, \tag{4}$$

where

$$\psi_i = 1 - \frac{1}{\sigma}\left[\frac{\phi\left(\frac{\mu_i}{\sigma} - \sigma\right)}{\Phi\left(\frac{\mu_i}{\sigma} - \sigma\right)} - \frac{\phi\left(\frac{\mu_i}{\sigma}\right)}{\Phi\left(\frac{\mu_i}{\sigma}\right)}\right] \tag{5}$$

and ϕ and Φ are the density and distribution functions of the standard normal variable, respectively. Appendix III shows the derivation of ψ_i. In accordance with Battese and Broca (1997), this refers to the first part of the expression in (4) as the elasticity of frontier output, and to the second part as the elasticity of technical efficiency. The latter part of these two components measures the proportion of the elasticity of mean output, which is due to a change in technical inefficiency.

Given the specification of models (1) and (2) for simultaneous estimation, this effect is non-zero only when the k-th variable is part-time helpers (k=3). In this case, the elasticity of technical efficiency is

$$-\psi_i\left(\frac{\partial \mu_i}{\partial Size_i}\right) = -\psi_i\left(\delta_2 + 2\delta_4 Size_i + \delta_6 Quality_i\right), \tag{6}$$

where $Size_i$=ln x_{3i}=ln z_{1i}. This expression is interpreted as the elasticity of technical efficiency with respect to provider size. The corresponding elasticity of technical efficiency with respect to service quality is

$$-\psi_i\left(\frac{\partial \mu_i}{\partial Quality_i}\right) = -\psi_i\left(\delta_3 + 2\delta_5 Quality_i + \delta_6 Size_i\right), \tag{7}$$

where $Quality_i$=ln z_{2i}. The elasticities defined by (6) and (7) are evaluated for different size and quality categories of providers in the following section in order to analyze the size-efficiency and quality-efficiency relationships. Returns to scale is defined as the sum of the elasticities of mean output with respect to all inputs.

2.2. DATA COLLECTION

The data set applied for the analysis in this paper is based on the results of a questionnaire survey on the current conditions of the *HLCS*. The survey was conducted by the research team working on long-term care insurance at Osaka School of International Public Policy, Osaka University in 2002. In order to send questionnaires to nation-wide home-visit long-term care providers, this paper accessed the social welfare and medical network system (WAM-NET) run by the Social Welfare and Medical Service (2002). WAM-NET lists information such as addresses, types of services, and corporate structures for currently existing long-term home-visit service providers in Japan. As of the time that the team conducted the survey, 15,000 home-visiting long-term care service

providers across the nation were registered on WAM-NET. This number is close to the total number of existing long-term home-visit service providers in Japan. The research team carried out a random sampling of 7,965 providers out of 15,000. [2] The respense rate was 16.3%. The questionnaires regarding service quality are based on Shimizutani and Suzuki (2002) and are listed in Appendix II. After data cleaning, 360 data were usable for the simultaneous estimations of both the stochastic frontier production model and the efficiency model.

2.3. EMPIRICAL RESULTS

Table 1 shows the descriptive statistics of our data. The general model defined by equations (1) and (2) is estimated using the methods of maximum likelihood. The para-

Table 1: Discriptive statistics for variables in the stochastic frontier model

	Sample mean	*Standard deviation*	*Minimum*	*Maximum*
Output	4042098.928	12242813.72	179472	199362000
Desk workers	2.03	8.59	1	160
Full-time Helpers	5.27	8.66	1	88
Part-time Helpers	18.86	42.02	1	401
Technology	2232.10	3192.02	121	18657
Quality	26.04	6.78	3	42

Note : All variables are not natural logarithms

2) Some for-profit service providers are franchisees of major service providers such as Comsn Community Medical System and Network or Human Life Care. Therefore, some franchise for-profit providers are not fully independent of each other by way of technical efficiency. It is likely that the franchise effect is positively related to technical efficiency. Unfortunately, our survey does not allow us to track down which providers are franchisees of which major service providers.

Table 2: Results from Estimation

Stochastic Production Frontier Model		coefficient (standard error)
Constant		13.828 ***
		(0.406)
Worker Dummy 1	D_1	0.028
		(0.095)
Worker Dummy 2	D_2	-0.454 ***
		(0.173)
Worker Dummy 3	D_3	-0.704 **
		(0.289)
Technology	$\ln x_{si}$	0.055
		(0.045)
Desk Workers	$\ln x_{1i}$	-0.0998
		(0.207)
Full-time Helpers	$\ln x_{2i}$	0.237 *
		(0.134)
Part-time Helpers	$\ln x_{3i}$	0.616 ***
		(0.149)
Desk Workers ^{2}	$\ln x_{1i}^2$	0.060
		(0.048)
Full-time Helpers ^{2}	$\ln x_{2i}^2$	0.045
		(0.043)
Part-time Helpers ^{2}	$\ln x_{3i}^2$	-0.062 *
		(0.032)
Desk Workers×Full-time Helpers	$\ln x_{1i} \times \ln x_{2i}$	-0.163 **
		(0.072)
Desk Workers×Part-time Helpers:	$\ln x_{1i} \times \ln x_{3i}$	0.040
		(0.043)
Full-time Helpers×Part-time Helpers	$\ln x_{2i} \times \ln x_{3i}$	0.085 ***
		(0.028)
Regional Dummy 1	DR_1	0.252 **
		(0.108)
Regional Dummy 2	DR_2	0.495
		(0.326)
Regional Dummy 3	DR_3	-0.054
		(0.128)
Organizational Dummy 1	DJ_1	0.968 ***
		(0.243)
Organizational Dummy 2	DJ_2	0.344
		(0.216)

Table 2: Results from Estimation (Continued)

Technical Inefficiency Model		coefficient (standard error)
Constant		-1.961 *
		(1.073)
Worker Dummy 3	D_3	-0.3298
		(0.473)
Size	$\ln z_{1i} (= \ln x_{3i})$	1.2853 ***
		(0.346)
Quality	$\ln z_{2i}$	1.716 **
		(0.761)
Size^2	$\ln z_{1i}^2$	-0.168 ***
		(0.060)
Quality^2	$\ln z_{2i}^2$	-0.347 **
		(0.141)
Size×Quality	$\ln z_{1i} \times \ln z_{2i}$	-0.188 **
		(0.092)
Organizational Dummy 1	DJ_1	1.038 ***
		(0.341)
Organizational Dummy 2	DJ_2	0.296
		(0.324)

Variance Parameters	coefficient (standard error)
$\sigma_s^2 = \sigma^2 + \sigma_v^2$	0.438 ***
	(0.041)
$\gamma = \sigma^2/(\sigma^2+\sigma_v^2)$	0.192 **
	(0.092)
Log-L	-350.98
Number of Observations	360
Mean Technical Efficiency	0.654

meter estimates of the stochastic frontier model are presented in Table 2.

For the translog function, the elasticities of mean output with respect to inputs, provider size, and service quality are functions of subsets of the parameters and the levels of the explanatory variables. Hence the individual coefficients in the stochastic frontier production function are not directly interpretable as elasticities, as for the Cobb–Douglas model.

Several generalized likelihood–ratio tests of null hypotheses involving restrictions on

Table 3: Generalized likelihood–ratio tests

Hull Hypothesis, H_0	*Log-Likelihood (LR) ratio test*	
Cobb-Douglas Form	$L(H_0)$	-358.379
number of restrictions = 6	$L(H_1)$	-350.982
	LR	14.793 ***
No inefficiency effects#	$L(H_0)$	-367.137
number of restrictions = 11	$L(H_1)$	-350.982
	LR	32.309 ***
No scale and quality	$L(H_0)$	-360.698
number of restrictions = 6	$L(H_1)$	-350.982
	LR	19.431 ***
No scale	$L(H_0)$	-353.781
number of restrictions = 4	$L(H_1)$	-350.982
	LR	5.597
No quality	$L(H_0)$	-368.068
number of restrictions = 3	$L(H_1)$	-350.982
	LR	34.171 ***
No organizational dummy	$L(H_0)$	-355.017
number of restrictions = 4	$L(H_1)$	-350.982
	LR	8.069 *
No regional dummy	$L(H_0)$	-355.166
number of restrictions = 3	$L(H_1)$	-350.982
	LR	8.368 **

#: According to Kodde and Palm (1986), the critical values of 5% significance level and 1% significance level are 19.045 and 24.049, respectively.
** and *** represent 5% significance level and 1% significance level, respectively.

the parameters in both the frontier and the inefficiency models are presented in Table 3. The first two tests consider the frontier function. Given the assumption of the tanslog stochastic frontier model, Cobb–Douglas technology (H_0: $\alpha_{kj}=0$) is rejected. The remaining

tests in Table 3 consider the restrictions on the parameters in the inefficiency model. The null hypothesis of no inefficiency effects is firmly rejected, and therefore, the average response function, in which all providers are assumed to be technically efficient, is not an adequate representation of the data given the assumption of the translog stochastic frontier model. This suggests that the maximum likelihood estimator is not equivalent to the ordinary least square estimator.

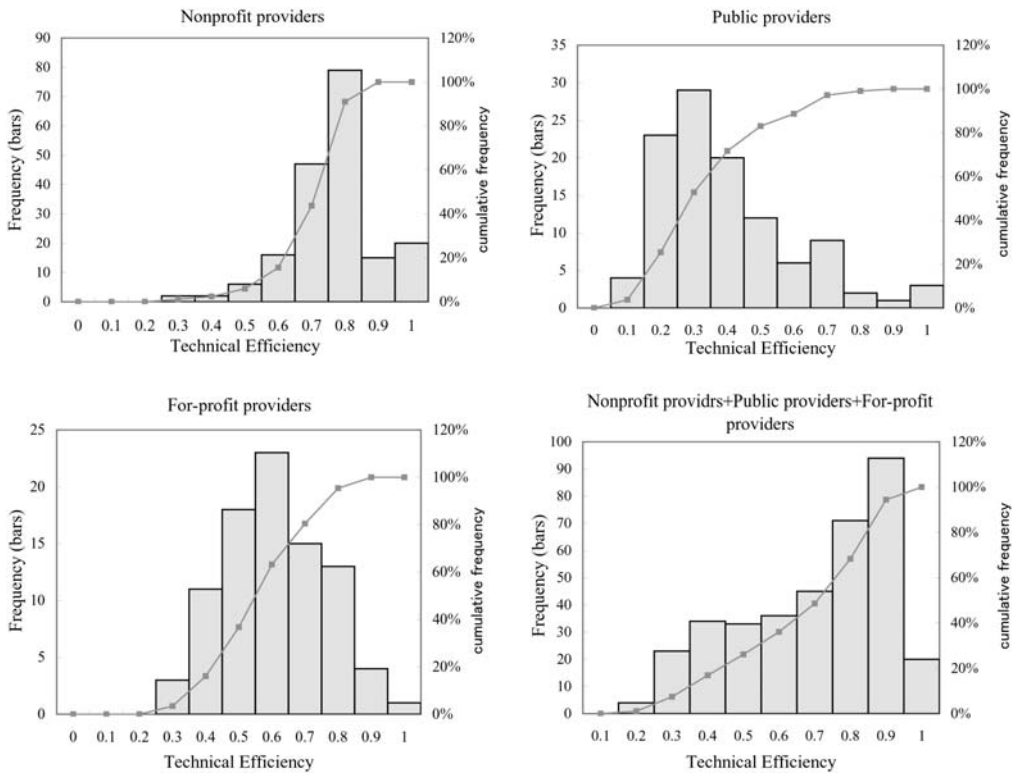
The estimate of γ $(=\sigma^2/\sigma_s^2 \in [0\ 1])$ = 0.192 implies that about 1/5 of the total variability is associated with the inefficiency effect, U_i and the rest is associated with the random error, V_i. Therefore, it appears that the stochastic frontier model is different from the deterministic frontier model.

The test of no service quality effect (H_0: $\delta_3=\delta_5=\delta_6=0$) can be rejected, whereas the test of no size effect (H_0: $\delta_1=\delta_2=\delta_4=\delta_6=0$) cannot. When the size and quality effects are tested jointly, the test (H_0: $\delta_1=\delta_2=\cdots=\delta_6=0$) can be rejected. Therefore, the conclusion in terms of an explanatory power requires careful consideration.

The test of no corporate structural difference both in production and technical inefficiency (H_0: $\beta_8=\beta_9=\delta_7=\delta_8=0$) is firmly rejected. The test of no regional difference in production can be also rejected. These tests suggest that the corporate structural differences of home-visit long-term care providers (nonprofit providers, the public providers, or for-profit providers) are significant both in production and inefficiency, whereas regional difference in production is not.

The distributions of the technical efficiencies of nonprofit providers, public providers,

Figure 2: Distributuions of technical efficiencies for home-visit long-term care providers



and for-profit providers are shown in Figure 2.

The distribution of the technical efficiencies of nonprofit providers is positively skewed, because a relatively large number of nonprofits is operating at high levels of technical efficiency. On the other hand, the distribution of technical efficiencies of public providers is negatively skewed. Thus, unlike nonprofit providers, few for-profit providers are operating at high levels of technical efficiency. However, the distribution of the technical efficiency of for-profit providers is neither negatively skewed nor positively skewed. The estimator for the mean technical efficiencies of the nonprofit providers, for-profit providers, and public providers calculated by *FRONTIER 4.1* are 0.8, 0.65, 0.43, respectively. This indicates that the nonprofit providers produce the *HLCS* most efficiently among the three. Therefore, nonprofit service providers are on average about 1.8 times more technically efficient than public providers. Likewise, for-profit providers are on average about 1.5 times more technically efficient than public providers. The tests of the mean differences of technical efficiencies among the nonprofit, for-profit, and public providers

3) We also compared Coefficient of Variations (*CV*) of technical efficiencies among nonprofits (*CV*=0.13), for-profits (*CV*=0.23), and public providers (*CV*=0.41) and found that the technical efficiencies of a relatively large number of pubic providers have deviated from the mean level. This deviation of government-managed organizations could be a mirror image of different management systems or production goals determined at the local governmental level.

show that their differences are statistically significant. [3] We may therefore ask what factors cause such differences among the three.

According to Hansmann (1980), nonprofits provide higher quality products than for-profit providers in a market with an information asymmetry between supplier and demander (market failure theory). While for-profit providers maximize profits by lowering the quality of services so that the costs of production decrease (opportunistic behavior), nonprofit providers by contrast do not have this opportunistic nature due to the non-distribution constraint. Property rights theory tells us that nonprofit providers tend to have profits not in the form of pecuniary benefits such as land and stock but in the form of non-pecuniary benefits since property rights are attenuated for nonprofit providers. (Williamson 1970; Alchian and Demsetz 1972; Frech 1976; Borjas and Ginsburg 1983) In our case, non-pecuniary benefits for nonprofit providers can be considered as the supply of high quality *HLCS* that meet the demands of non-median voters. Property rights are also attenuated for public providers. However, public providers tend to satisfy only the demands of median voters. Consequently, service quality of for-profit providers is expected to be lower than that of nonprofit providers.

However, this scenario might be acceptable only in the short run because, in the long run, service consumers will have chances to observe the quality of services more closely. If the market failure theory holds in the long run, then all for-profit providers should lose their customers and eventually they should exit from the market, In reality, however, all three types of service providers exist in the market, such as the US nursing home market and the Japanese *HLCS* market. Therefore, it is conceivable that the market failure theory may not be supportable in the long run and information asymmentry in terms of service quality may disappear in the long run. The tests of mean differences of service quality among the three in this paper imply that there exist no statistically significant mean differences of service quality among the nonprofit, the for-profit, and the public providers. [4] Thus, we may conclude that *HLCS* quality difference is not the factor causing differences in technical efficiency. [5]

Then, we may ask what else could cause such differences among the three One factor could be the difference of labor efficiency. As labor efficiency is not rewarded in public enterprises due to attenuated property rights. This poor labor efficiency could have an adverce impact on technical efficiency. The level of labor efficiency and wage rates in nonprofit organizations are not closely connected either. Hence, we would expect labor

4) Previous research shows that whether nonprofits or for-profits supply the higher quality of production is inconclusive. Nyman (1988) and O'Brien et al. (1983) found that the quality of production between for-profits and nonprofits is the same, while Ullmann and Holtmann (1985), Cohen and Spector (1996) found that nonprofits supply higher quality of production than for-profits and Gertler (1989) concludes that for-profits supply a higher quality of production than nonprofits.

5) It has been three years since the long-term care insurance system was enacted. There is still discussion as to whether three years should be considered within the long run or the short run.

efficiency in nonprofits not to raise wage rates. However, the workers' objective is to obtain more non-pecuniary benefits, which should ameliorate the workers' effort level in nonprofits. On the other hand, since labor efficiency and wage rates are deeply connected, as in for-profits, greater labor efficiency effort should ameliorate the technical efficiency of production. Consequently, the mean technical efficiency in public providers may result in the lowest among the three.

Table 4: Elasticities and Returns to scale

Desk workers	-0.1810
Full-time helpers	0.1409
Part-time helpers	0.4576
Returns to scale	0.4175

Note: All elasticities are mean input levels

Elasticity of mean output with respect to the three input variables is shown in Table 4. The elasticity for part-time helpers is higher than those for desk workers and full-time helpers. The estimate for the elasticity for desk workers is negative, showing that the dramatic output decrease may have been associated with over-hiring desk workers who do not directly provide home-visiting long-term care services. [6] The returns to scale are approximately 0.4, indicating home-visiting long-term care service production has decreasing returns to scale.

We follow Lundvall and Battese (2000) to investigate the effect of growth in provider size on technical efficiency in consideration of interactions between the provider size and service quality. In order to investigate the sign of the size-efficiency effect, we need to evaluate the partial derivative of the mean of the inefficiency effects, μ_i with respect to

$$\frac{\partial \mu_i}{\partial Size_i} = \delta_2 + 2\delta_4 Size_i + \delta_6 Quality_i. \tag{8}$$

size. Then the size-efficiency effect is given by

The sign of the derivative of equation (8) depends not only on the sign on parameters involved, but also on the values of the size and quality variables. In fact, the expression in (8) shows the marginal effect of size is a linear function of size and quality. By setting the

$$Size_i = -\left(\frac{\delta_2 + \delta_6 Quality_i}{2\delta_4}\right), \tag{9}$$

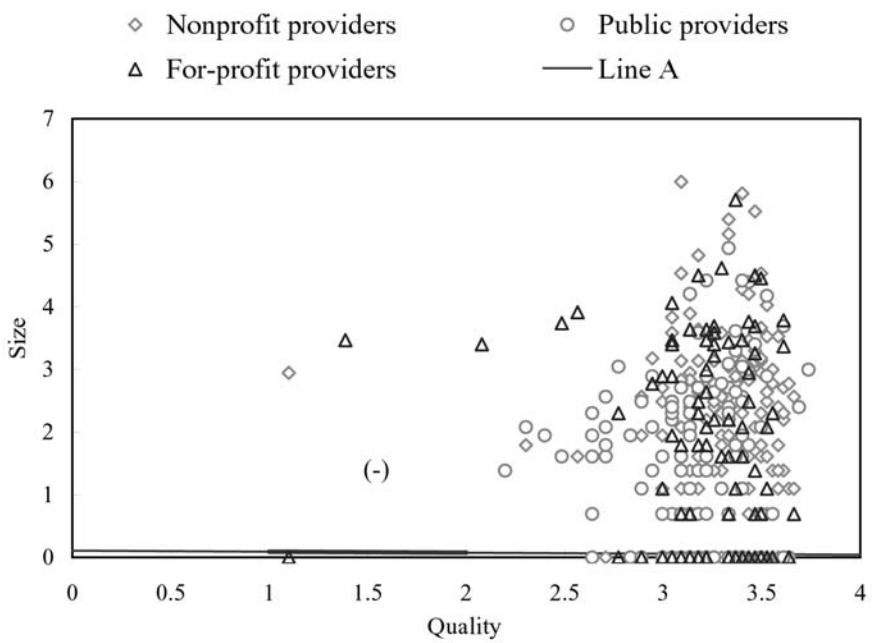
derivative in (8) to zero and solving for $Size_i$, we obtain

which is the Line A in the size-quality space in Figure 3. This line defines the set of com-

6) The negative elasticity of frontier output for desk workers indicates that the usage of desk workers is not optimal and/or the frontier model estimated above could not be economically satisfactory. In either case, the frontier elasticity for desk workers estimated above should not be referred to as being associated with "best practice" home-visit long-term care service production.

binations of size and quality for which the marginal effect of size on technical inefficiency is zero. On one side of the line, the effect of size is either positive or negative. In Figure 3, the size–quality values for all observations are plotted together with Line A. An overwhelming majority of providers are on that side of the line where the marginal effect of size on technical inefficiency is negative (−) since $\partial(\partial\mu_i/Size_i)/\partial Quality_i = \delta_6 < 0$ holds. This implies that the size–efficiency relationship is positive, and technical efficiencies of a majority of providers improve as the size of home–visit long–term care service providers

Figure 3: The marginal effect of firm size on technical inefficiency



grows.

On the other hand, in order to investigate the sign of the quality–efficiency effect, we

$$\frac{\partial\mu_i}{\partial Quality_i} = \delta_3 + 2\delta_5 Quality_i + \delta_6 Size_i. \tag{10}$$

need to evaluate

The expression in (10) shows the marginal effect of quality is a linear function of size and

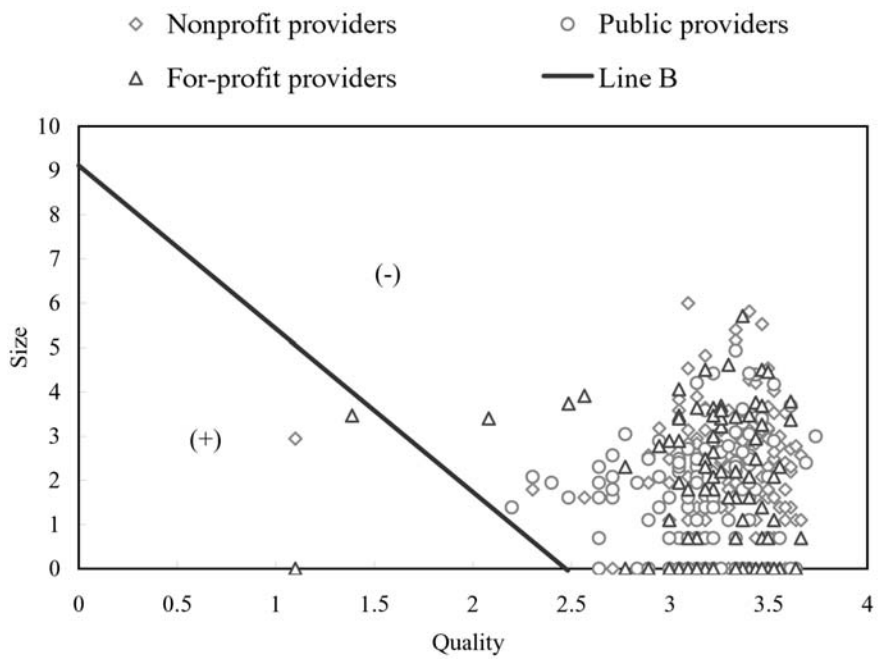
$$Size_i = -\left(\frac{\delta_3 + 2\delta_5 Quality_i}{\delta_6}\right), \tag{11}$$

quality. By setting the derivative in (10) to zero and solving for $Size_i$, we obtain

which is the Line B in the size–quality space in Figure 4. Line B defines the size–quality combinations for which the quality effects are zero. As we can see from Figure 4, all but three providers are on the side of the line where the marginal effect of service quality on

technical inefficiency is negative (−) because $\partial(\partial\mu_i/\partial Quality_i)/\partial Quality_i = 2\delta_5 < 0$ holds. This implies that the quality–efficiency relationship for all but three providers is positive (+), and technical efficiencies of providers tend to improve as the quality of ser-

Figure 4: The marginal effect of service quality on technical inefficiency



vice becomes higher.

The absolute value of Line B's slope is relatively steeper than that of Line A's slope. This implies that impact of the service quality on the scale–efficiency relationship is greater than that of the firm size on the quality–efficiency relationship.

3. CONCLUSIONS

This paper applied the stochastic frontier production model in order to explore the relationships between technical efficiency and provider size, and between technical efficiency and service quality, in the mixed economy of Japan. The analysis revealed that, given the specifications of the translog stochastic frontier production model, the Cobb–Douglas functional form is not an adequate representation of the data. Further, the technical inefficiency of home–visit long–term care service production was significantly related to the various explanatory variables considered in this paper. We note that, if these features are not accounted for in the frontier modeling, then conclusions may be inappropriately made. We also note that our analysis does not exclude the possibility that quality of *HLCS* may

have a 'lumpy' distribution by its nature since it is very difficult to measure and has information asymmetry.

Based on the model specification in this paper, another finding was that the mean technical efficiency generally increased with service quality. Also, hypothesis testing revealed that provider size might have no explanatory power. However, it should be noted that this is sensitive to how we define provider size.

A caveat in this paper is that the model for estimation does not contain the explanatory variable for physical capital because it could be one of the important factors for production of Home-visit Long-term Care Services. The problem comes from the lack of data for physical capital due to the fact that very few respondents filled out the questionnaire asking how much physical capital they possess. However, it is conceivable that the production of Home-visit Long-term Care Services should be highly labor intensive because helpers' main jobs are housekeeping chores. Therefore, it is highly unlikely that the luck of physical capital data may undermine the credibility of analyses in this paper.

The analysis in this paper also revealed that among the three categories of nonprofit providers, public providers, and for-profit providers, a relatively large number of nonprofit providers produce the *HLCS* efficiently, while a relatively large number of public providers produce them less efficiently.

For policy considerations, this study suggests that both growth in size and improving service quality may lead to high technical efficiency. Support programs for these two factors should thus be designed in order to provide an environment of service production that stimulates growth in size and the improvement of service quality.

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Appendix I: Regional Division

Special area

[Metropolis of Tokyo]

Nerima City, Nakano City, Shinnbashi City, Suginami City, Setagaya City, Kita City, Toyoshima City, Shinjiuku City, Shibuya City, Meguro City, Adachi City, Arakawa City, Bunkyo City, Chiyoda City, Chuo City, Minato City, Shinagawa City, Ota City, Daito City, Sumida City, Eto City, Katushika City, Edogawa City

Special area A

[Metropolis of Tokyo]

Hachioji City, Tachikawa City, Musashino City, Mitaka City, Fucyu City, Syowajima City, Cyofu City, Machida City, Koganei City, Kodaira City, Hino City, Higashimurayama City, Kokubunji City, Kunitachi City, Nishitokyo City, Komae City, Tama City, Inagi City

[Kanagawa Prefecture]

Yokohama City, Kawasaki City, Yokosuka City, Kamakura City

[Aichi Prefecture]

Nagoya City

[Kyoto Prefecture]

Kyoto City

[Osaka Prefecture]

Osaka City, Sakai City, Kishiwada City, Toyonaka City, Ikeda City, Suita City, Izumiootsu City, Takatuki City, Moriguchi City, Hirakata City, Ibaraki City, Yao City, Neyagawa City, Matsubara City, Daito City, Izumi City, Mino City, Kadoma City, Settsu City, Takaishi City, Higashiosaka City, Shijyonawate City, Katano City, Tadaoka town

[Hyogo Prefecture]

Kobe City, Amagasaki City, Nishinomiya City, Ashiya City, Itami City, Takarazuka City, Kawanishi City

Area A

[Kanagawa Prefecture]

Izu City, Hayama Town

[Osaka Prefecture]

Kaizuka City, Izumisano City, Tondabayashi City, Kashihara City, Habikino City, Fjiidera City, Osakasayama City, Shimamot Town, Kumadori Town, Mihara Town

[Fukuoka Prefecture]

Kitakyushu City, Fukuoka City

Area B

[Hokaido]

Sapporo City, Otaru City,

[Miyagi Prefecture]

Sendai City

[Saitama Prefecture]

Kawagoe City, Kawaguchi City, Saitama City, Tokorozawa City, Iwaki City, Kusaka City, Warabi City, Koshigaya City, Toda City, Hatogaya City, Asaka City, Shiki City, Wako City, Niiza City, Fujimi City, Kamifukuoka City, Oi Town, Miyoshi Town

Area B (continued)

[Chiba Prefecture]

Chiba City, Ichikawa City, Funabashi City, Matsudo City, Kashiwa City, Furayasu City, Yotsukaido City

[Metropolis of Tokyo]

Oume City, Fussa City, Higashiyamato City, Kiyose City, Higashikurume City, Musashimurayama City,

[Kanagawa Prefecture]

Hiratsuka City, Fujisawa City, Odawara City, Chigasaki City, Sagamihara City, Miura City, Atsugi City, Yamato City, Isehara City, Ebina City, Zama City, Ayase City, Samukawa Town

[Shizuoka Prefecture]

Shizuoka City, Atami City, Ito City

[Shiga Prefecture]

Otsu City

[Kyoto Prefecture]

Uji City, Mukou City, Nagaokakyou City

[Osaka Prefecture]

Kawachinagano City, Izumiminami City, Hannan City, Tajiri Town

[Hyogo Prefecture]

Himeji City, Akashi City, Mita City

[Nara Prefecture]

Nara City, Yamatokoriyama City, Ikoma City

[Wakayama Prefecture]

Wakayama City

[Okawama Prefecture]

Okayama City

[Hiroshima Prefecture]

Hiroshima City

[Yamaguchi Prefecture]

Shimonoseki City

[Fukuoka Prefecture]

Kurume City, Iizuka City

[Nagasaki Prefecture]

Nagasaki City

Other Area

Cities, Towns, and, Villages not listed in Special wards, Special Area, Area A, and Area B.

The data in this survey relates to the regional divisions as they were the data was collected in 2003.Note that some regional divisions have changed since that time.

Appendix II: Indexes of service quality

Service management (4 points)	Service manuals have been drawn up Care conferences and meetings to discuss care plans were carried out Reports were written after care services Records of care services and consultations were kept
Training system (4 points)	Staff trainings executed at time of employment Dispatched employees to outside training run by public associations and associations of helpers Regular staff trainings executed OJT inside the company has been carried out
Health and anti-infection measures (4 points)	Is instructing employees how to prevent infectious diseases Is regulating employees' clothing Hygienists and registered nurses are regularly sent to service users for health checks A person in charge of hygienic management and infectious diseases exists
Information offer and troubleshooting (4 points)	Has user support centers Has complaint handlers Brochures and/or websites introducing contents of providable services exists Is possible to try services
Supply systems of services (4 points)	Always visits users and draws out home-visit long-term care service plans Consults regularly with users' families and hears their opinions Induces self-support of users by using diapers on minimum basis, increasing the time out of bed, and doing rehabilitation exercises Asks opinions of hygienists, registered nurses, and/or doctors in the making of a care service plan
Convenience of services (4 points)	Able to supply early-morning services Able to supply late-night services Services outside the care plan are available at time of emergency Is open on Sundays and national holidays
Correspondence in case of accident or emergency (4 points)	Doctors and medical facilities on a short-term contract are available Manuals in case of emergency or accident are prepared Possesses damage insurance in case of emergency A person is in charge of emergency situations or accidents.
Situation of settlement of business plan (4 points)	Business concepts and/or business policies are written out and made clear Business plans are made and written out Budget plans are made and written out Financial statements are audited by external auditors
Personnel management (4 points)	Company regulations exist Executes employee evaluations and performance evaluations Work distribution is managed System to assist employees in obtaining qualifications exists
Privacy protection (4 points)	A person in charge of keeping users' logs exists Keeps users' logs in storage facilities with security. Is regulating duty of confidentiality against workers Is educating employees about human rights

Appendix III: Calculation of Elasticities of Mean Output

The stochastic frontier model can be expressed by

$$y_i = \exp(x_i\beta)\exp(V_i - U_i), \tag{A1}$$

where V_is are $iid\ N(0, \sigma_v^2)$, independent of the U_is which are independently distributed, such that U_i is obtained by the truncation (at zero) of the $N(\mu_i, \sigma^2)$, where μ_i is defined by

$$\mu_i = z_i\delta_i, \tag{A2}$$

and z_i may contain some variables in x_i.

Given the assumption about the distribution of V_i and U_i, we note as follows.

Note 1: The density function for U_i is

$$f_{U_i}(u) = \frac{1}{\sqrt{2\pi}\sigma}\frac{\exp\left[-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2\right]}{\Phi(\mu/\sigma)}, \quad u \geq 0 \tag{A3}$$

where and $\Phi(\theta) = \int_{-\infty}^{\theta}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\omega^2\right)d\omega.$

Note 2: If $S \sim N(\mu, \sigma^2)$, then $P(S > 0) = 1 - \Phi\left(\frac{-\mu}{\sigma}\right) = \Phi\left(\frac{\mu}{\sigma}\right)$.

Note 3: For $V_i \sim N(0, \sigma_v^2)$, $E(e^{v_i}) = \exp\left(\frac{1}{2}\sigma_v^2\right)$, which can be considered as the mean of the lognormal distribution.

Given the above results, it follows that

$$E[y_i] = \exp(x_i\beta)\exp\left(\frac{1}{2}\sigma_v^2\right)E[\exp(-U_i)]. \tag{A4}$$

Now $E[\exp(-U)] = \int_0^\infty \exp(-u)\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2\right]\frac{1}{\Phi(\mu/\sigma)}du$, (A5)

where i is omitted for simplicity. In the integral above, consider the exponent,

$$\kappa = -u - \frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2.$$

Then,
$$\begin{aligned}\kappa &= -\frac{1}{2\sigma^2}\left[u^2 - 2u(\mu-\sigma^2) + \mu^2\right] \\ &= -\frac{1}{2\sigma^2}\left[(u-(\mu-\sigma^2))^2 + \mu^2 - (\mu-\sigma^2)^2\right] \\ &= -\frac{1}{2}\left\{\left[\frac{u-(\mu-\sigma^2)}{\sigma}\right]^2\right\} - \mu + \frac{1}{2}\sigma^2.\end{aligned}$$

Therefore, equation (A5) can be written as

$$E[\exp(-U)] = \int_0^\infty \exp\left(-\mu+\frac{1}{2}\sigma^2\right)\frac{1}{\sqrt{2\pi}\sigma}\frac{\exp\left(-\frac{1}{2}\left[\frac{u-(\mu-\sigma^2)}{\sigma}\right]^2\right)}{\Phi(\mu/\sigma)}du. \quad \text{(A6)}$$

Let $\varsigma = \frac{u-(\mu-\sigma^2)}{\sigma}$, then

$$\begin{aligned}E[\exp(-U)] &= \frac{\exp\left(-\mu+\frac{1}{2}\sigma^2\right)}{\Phi(\mu/\sigma)}\int_{-\frac{(\mu-\sigma^2)}{\sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\varsigma^2\right)d\varsigma. \\ &= \frac{\exp\left(-\mu+\frac{1}{2}\sigma^2\right)\left[1-\Phi\left(-\frac{\mu}{\sigma}+\sigma\right)\right]}{\Phi(\mu/\sigma)} \\ &= \exp\left(-\mu+\frac{1}{2}\sigma^2\right)\left[\frac{\Phi\left(\frac{\mu}{\sigma}-\sigma\right)}{\Phi\left(\frac{\mu}{\sigma}\right)}\right].\end{aligned}$$

Thus, $\ln E(y) = x\beta + \frac{1}{2}\sigma_v^2 - \mu + \frac{1}{2}\sigma^2 + \ln\left[\Phi\left(\frac{\mu}{\sigma}-\sigma\right)\right] - \ln\Phi\left(\frac{\mu}{\sigma}\right)$ holds. Finally, it can be shown that the elasticity of mean output with respect to input k is

$$\frac{\partial \ln E(y)}{\partial x_k} = \frac{\partial x\beta}{\partial x_k} + \frac{\partial \mu}{\partial x_k}\left\{-1 + \frac{1}{\sigma}\left[\frac{\phi\left(\frac{\mu}{\sigma} - \sigma\right)}{\Phi\left(\frac{\mu}{\sigma} - \sigma\right)} - \frac{\phi\left(\frac{\mu}{\sigma}\right)}{\Phi\left(\frac{\mu}{\sigma}\right)}\right]\right\}, \tag{A7}$$

where $\frac{\partial \Phi(\theta)}{\partial \theta} = \phi(\theta)$. Note that if μ does not contain x_k, then the second term of the right-hand side of equation (A7) is zero. As we replace non-stochastic variables $x_k s$ by $\ln x_k s$, we obtain equations (4) and (5).